Fuzzy Components in Community Level Comparisons

V. De Patta Pillar and L. Orlóci

Department of Plant Sciences, University of Western Ontario, London, Ontario, Canada N6A 5B7

Abstract

The conventions of traditional plant community analysis assume an arrangement of the community components in discrete populations. While this assumption is in line with the notion of absolute discreteness in classical taxonomy in the sense that no organism may belong to more than one taxon, it is not on all fours with reality. The assumption of overlapping populations is indeed more realistic by virtue of the fact that organisms will show affinities to other organisms not in their taxa and not in their community. Starting from this fundamental fact, and observing that by practicing the assumption of absolute discreteness leads to an over-abundance of zeros (absences) in the data and through this to increased levels of indeterminacy in community comparisons, it makes sense to us to pass from an absolutely discrete taxonomy to one which has provisions for similarity-based population overlap. In one case, the field records use discrete taxa which are replaced in the analysis by their fuzzy set equivalents; the latter which emphasize *a posteriori* taxon similarities. Fuzzy sets of this kind form the basis of a new similarity measure which we propose for community level comparisons. We describe this measure and illustrate it by example.

The problem

For purposes of study, the *unit plant community* and the *community component* have to be defined. This may seem a trivial matter, but it is not, considering that the unit plant community is an areal dissection of the landscape and the community components are dissections of the organismal multitude. While the choice of area size/shape has to be linked with sampling considerations (Orlóci and Pillar 1989 and Chapter 4), the dissection of the organismal multitude requires a taxonomic scheme. Traditionally this scheme uses common inheritance as its defining criteria for taxa in which case species populations are recognized. Other schemes better suited for specific ecological work use common survival characteristics which give rise to a taxonomy of Character Set Types (CSTs; Orlóci 1988, 1991). Inherent in the definition of the unit community and the community component are scale (area) and context (taxonomy) which delimit community studies.

The taxonomic scheme of particular interest to us is an *intersection taxonomy* (Orlóci 1988, 1991) which comes about when the taxonomy of the defining characters intersects (in a score matrix) the taxonomy of the organismal populations. An elementary consequence of this is readily apparent in the unit community's phytosociological relevé which becomes a matrix of character scores and CST cover/abundance (C/A) estimates. It is important to note that the different taxonomies dissect the multitude of community components differently. Through this, they allow different numbers of absences (zeros) to occur in the records which generate different levels of indeterminacy in community comparison. We suggest that in this regard CST-based community components will likely be less frequently absent than the species based components and consequently the absence-related indeterminacy will be lower. We measure an absence-related indeterminacy in terms of the number of community components shared by two communities α and β:

$$\text{Indeterminacy}_{\alpha,\beta} = 1 - \frac{\text{number of shared components}}{\text{total number of distinct components in } \alpha \text{ and } \beta}$$

This increases as the number of components not shared increases. Total indeterminacy occurs when no community component is shared. Thus indeterminacy will set the limits of the universe within which community comparisons are meaningful. A species-based taxonomy narrows these limits to the same type of habitat within a floristic

E. Feoli and L. Orlóci (eds.), Computer Assisted Vegetation Analysis, 87–93.

region. A CST-based taxonomy is likely to broaden the limits.

One of the fallacies in community studies is the assumption that the community components are absolutely discrete populations. This is to be understood in the sense that population membership has no overlap. An analysis which accepts this assumption is vulnerable, since it treats all absences (zeros) as denoting the same, no matter how similar the absent components are to any of those that are present. To overcome this problem, it is suggested to view community components as fuzzy entities and to represent them in the analysis by *fuzzy sets*. In these terms the organism can no longer be identified as an exclusive member of one or another taxon, but as an entity with membership in each of several taxa, albeit their membership may not be equally strong. All taxa have membership values in all of the others. Any such value expresses a taxon's affinity to another on a 0 to 1 scale. Affinity in our case is symmetric. The set of these values for a given taxon is the taxon's associated *fuzzy set*. Fuzzy sets have been applied in vegetation ecology in typification (Feoli and Zuccarello 1986, 1988, Marsili-Libelli 1989, Dale 1988) and in ordination (Roberts 1986), but these applications have considered fuzziness at the community level and not at the community component level.

How to incorporate the community components' fuzziness into the resemblance measure when comparing communities is the main problem. What gives significance to this is that comparisons are very basic in ecological reasoning, and of course, in the practice of data manipulations. Any device that enhances community level comparability should therefore be welcome.

Population (CST) level

We identify community components as CSTs and we describe them by a hierarchical character set. The hierarchical arrangement is predetermined. Since the character set is the same, the shared CSTs of communities can be recognized. Furthermore, the degree of overlap can be defined by considering partially merged runs of different CSTs through the character hierarchy which reflect their similarities. Thus the problem of constructing a fuzzy set equivalent of each CST can be handled on the basis of the degree of similarity between the CSTs. It should, however, be mentioned that our construction technique is different from others in which fuzzy sets are based on the degree of compositional similarity between entire stands bypassing the population level.

Since the CSTs are mapped into a character hierarchy as runs through the nodes, CST similarities will change depending on the hierarchic level. The ith of m levels contains k_i nodes. At a node d there is defined a vector $\mathbf{v}_{id}$ composed of the states of the first $m - i + 1$ characters in the set (the ith level plus the $m - i$ higher levels in the hierarchy). The set of CSTs in the relevé score matrix corresponds to the realized $\mathbf{v}_{1d}$ vectors, while, e.g., the $\mathbf{v}_{2d}$ vectors correspond to partially merged CSTs; the merging is the consequence of the character on level 1 (last in the character set) not being considered. We define the fuzzy set associated with any $\mathbf{v}_{id}$ as the relation,

$$F_{id} = \{\mathbf{v}_{ie}, u_{ie,id}: \mathbf{v}_{ie} \text{ similar to } \mathbf{v}_{id}\} \quad e=1, \ldots, k_i$$

That is, on level i at node e the vector $\mathbf{v}_{ie}$ has membership value ($u_{ie,id}$) in the fuzzy set composed of the vectors similar to the vector $\mathbf{v}_{id}$ at node d. The membership value ($u_{ie,id}$) is measured by any similarity index (S_{ide}) in the interval [0, 1], such as the Gower index (1971) in which we include a weighting factor[1]. Just as the CSTs have C/A values, the associated fuzzy sets also have C/A values. For fuzzy set F_{id} in community α, the C/A value is

$$Y_{id\alpha} = 0 \text{ if } \sum_{\beta=1}^{v} X_{id\beta} = 0 \tag{1a}$$

or as an alternative rule

$$Y_{id\alpha} = 0 \text{ if } X_{id\alpha} = 0 \text{ and } X_{id\beta} = 0 \tag{1b}$$

If neither condition is true then

$$Y_{id\alpha} = \sum_{\beta=1}^{k_i} \left[(u_{ie,id})^{1/\xi} X_{ie\alpha} \right] \tag{2}$$

for any $i = 1, \ldots, m$ level, $d = 1, \ldots, k_i$ node, and $\alpha = 1, \ldots, v$ community. $X_{ie\alpha}$ is the cumulative cover-abundance at level i, node e, community α. Note that ξ is an arbitrarily set fuzziness level always larger than 0 but not greater than 1. Rules 1a and 1b are alternatives. Instead of using rule 1a, which assumes that the CST for which the fuzzy set F_{id} is defined is present in at least one community in the collection to be considered as potentially present in community α, an alternative approach uses rule 1b for the community pair α, β. In this case, different cover-abundances arise for each community pair. In either case in order not to change the total cover-abundance, a correction is applied:

$$Z_{id\alpha} = Y_{id\alpha} \frac{\sum_{e=1}^{k_i} X_{ie\alpha}}{\sum_{e=1}^{k_i} Y_{ie\alpha}}$$

Community level

We use the Z values of (3) to measure the resemblance of community α to β. On the ith level of the character hierarchy we define a nominal product of the descriptors of α and β:

$$Q_{i\alpha\beta} = \sum_{d=1}^{k_i} \left[n_i \left(\frac{Z_{id\alpha}}{n_i} - \bar{Z}_{i\alpha} \right) \left(\frac{Z_{id\beta}}{n_i} - \bar{Z}_{i\beta} \right) \right]$$

We do the same for all $v(v-1)$ distinct community pairs, which is analogous to the case described in Orlóci and Orlóci (1985). In the above equation, k_i represents the number of nodes at level i, n_i the number of 1st order CSTs (level 1) fused at any node on level i, and the $\bar{Z}_{1s}$ the mean cover-abundances of the fuzzy sets at the first level in $s = \alpha$ or β.

Example

The data set is from sub-boreal vegetation on our Elk Lake recovery site in Ontario, Canada. The vegetation is secondary, 3 years after logging. The vegetation description is by score matrix relevés. The CSTs are the community components. The character set is listed in Table 1 and a partial data set is given in Table 2. To give an example, the fuzzy set equivalent of CST 5 (row 5 in Table 2) on hierarchy level 1 (i.e., considering all 13 characters) has 27 elements:

$F_{1,5}$ {(1, 0.404), (2, 0.462), (3, 0.243), (4, 0.223), (5, 1), (6, 0.196), (7, 0.338), (8, 0.223), (9, 0.377), (10, 0.281), (11, 0.262), (12, 0.769), (13, 0.185), (14, 0.146), (15, 0.296), (16, 0.219), (17, 0.319), (18, 0.35), (19, 0.277), (20, 0.338), (21, 0.269), (22, 0.223), (23, 0.358), (24, 0.392), (25, 0.315), (26, 0.408), (27, 0.365)}

In this the 5th CST vector $\mathbf{v}_{1,5}$ = (9 6 6 2 1 3 1 1 1 1 1 1 1) (row 5, Table 2) is compared to 26 other CSTs and itself. Each comparison is specified by two numbers, say (4, 0.223). The 1st element in the pair identifies the 4th CST vector $\mathbf{v}_{1,4}$ (1 39 3 4 2 1 3 2 1 4 3 1 3) (row 4, Table 2) the elements of which represent character states over 13 characters (e.g., state 3, last character). The 2nd element (0.223) is a measure of the membership value of the 4th CST in the fuzzy set equivalent of the 5th CST. The following scheme on page 90 illustrates the computation of a membership value, involving the 4th and 5th CST vectors (Table 2).

Table 1. Character set used in community description on the Elk Lake transect site. References identify only the general class of notions. The system adopted may differ. 'Stem' and 'leaf' refers to stem and stem-like or leaf and leaf-like structures.

Life-form (Mueller-Dombois & Ellenberg 1974)

1. Form. 1: Phanerophytes, 2: Chamaephytes, 3: Hemicryptophytes, 4: Geophytes, 5: Therophytes, 6: Lianas, 7: Hemi-epiphytes, 8: Epiphytes, 9: Thallo-chamaephytes, 10: Thallo-hemicryptophytes, 11: Thallo-therophytes, 12: Thallo-epiphytes

Growth-form (Barkman 1988)

2. Form. 1: stereocaulid, 2: cladinid, 3: peltigerid, 4: usneid, 5: sphagnid, 6: polythrichid, 7: hylocomniid, 8: pleuroziid, 9: rhacomitriid, 10: hypnid, 11: climaciid, 12: myriophyllid, 13: convolvulid, 14: smilacid, 15: equisetoid, 16: reptant graminid, 17: stoloniferous graminid, 18: rhizomatous graminid, 19: hemicaespitose graminid, 20: caespitose graminid, 21: solitary graminid, 22: pulvinate herb, 23: decumbent herb, 24: arching herb, 25: rosulate herb, 26: scapos-rosulate herb, 27: erect scapose herb, 28: erect epirosulate, 29: oxycoccid shrub, 30: arctostaphyllid shrub, 31: andromedid (vacciniid), 32: juniperid, 33: rubid, 34: sambucid, 35: cornid, 36: cornoflorid, myricid (sarothamnid), 37: cupressoid, 38: piceid, 39: betulid, 40: fagid, 41: else

Stem (Shreve 1942)

3. Type. 1: short caudex, 2: long caudex, 3: truncas, 4: caulis, 5: culm, 6: stipe, 7: no stem
4. Consistence. 1: succulent, 2: herbaceous, 3: semi-ligneous, 4: ligneous, 5: no stem
5. Direction. 1: erect, 2: erect diffuse, 3: creeping, 4: climbing, 5: no stem

Leaf (Dansereau 1957)

6. Type. 1: deciduous, 2: withering, 3: evergreen, 4: no leaf
7. Shape. 1: needle, 2: graminoid, 3: broad, simple, 4: compound, 5: thalloid, 5: no leaf
8. Texture. 1: filmy, 2: membranous, 3: sclerophyll, 4: succulent, 5: no leaf

Leaf (Shreve 1942)

9. Epidermal surface. 1: glabrous, 2: glaucous, 3: trichomous dense, 4: trichomous sparse, 5: no leaf

Leaf (Orlóci and Orlóci 1985)

10. Width. 1: < 2.5 mm, 2: 2.5–5, 3: 5–10, 4: 10–50, 5: 50–100, 6: 100 <:, 0: no leaf
11. Length. 1: < 5 mm, 2: 5–25, 3: 25–75, 4: 75–125, 5: 125 <, 0: no leaf
12. Thickness. 1: < 1 mm, 2: 1–3, 3: 3–5, 4: 5 <, 0: no leaf

Plant height (Dansereau 1957)

13. Height class. 1: < 0.1 m, 2: 0.1–0.5, 3: 0.5–2, 4: 2–8, 5: 8–10, 6: 10–25, 7: > 25 m

Table 2. CST cover-abundances (C/A) in 9 quadrats on the Elk Lake recovery transect site. The CSTs are defined by the states of the 13 characters shown in Table 1. The last two columns contain the pairwise-adjusted cover-abundance values according to rule 1b (see the main text) of quadrats 8 and 9, hierarchy level 1 with 27 realized nodes (CSTs). The weight is 1 for all characters and the degree of fuzziness is also 1. As an example, the cover-abundance of fuzzy set $F_{1,5}$ in quadrat 8 is obtained as follows:

$F_{1,5}$ {(1, 0.404), (2, 0.462), (3, 0.243), (4, 0.223), (5, 1), (6, 0.196), (7, 0.338), (8, 0.223), (9, 0.377), (10, 0.281), (11, 0.262), (12, 0.769), (13, 0.185), (14, 0.146), (15, 0.296), (16, 0.219), (17, 0.319), (18, 0.35), (19, 0.277), (20, 0.338), (21, 0.269), (22, 0.223), (23, 0.358), (24, 0.392), (25, 0.315), (26, 0.408), (27, 0.365)}. This is explained in the main text.

$Y_{1,5,8} = (0.1)(0.223) + (0)(1) + (0.01)(0.223) + (0.1)(0.281) + (3)(0.296) + (0.1)(0.277) + (0)(0.338) + (0.01)(0.315) + (0.1)(0.408) = 1.012$. This is based on equation (2). The other Y values needed in the sum to calculate the Z values are not given here.

$Z_{1,5,8} = (1.012)\ [(0.1 + 0.01 + ... + 0.1)/(2.556 + 1.012 + 1.340 + 2.998 + 3.257 + 2.046 + 1.444 + 0.948 + 0.906)] = (1.012)(3.42)/(16.508) = 0.21$. This is based on equation (3).

Characters 1 2 3 4 5 6 7 8 9 10 11 12 13	Unadjusted C/A values in quadrats									Fuzzy-adjusted C/A values	
CSTs	1	2	3	4	5	6	7	8	9	$Z_{1\alpha 8}$	$Z_{1\alpha 9}$
1 38 3 4 1 3 1 3 2 1 2 2 2	0	0.01	0	0	0.1	0	0	0	0	0.00	0.00
1 38 3 4 1 3 1 3 1 1 3 2 2	0.1	0.1	0.1	1	1	0	1	0	0	0.00	0.00
1 39 3 4 2 1 3 2 1 4 2 1 3	2	2	1	0	1	0	0	0	0	0.00	0.00
1 39 3 4 2 1 3 2 1 4 3 1 3	0.01	0.1	0	2	1	2	2	0.1	0.1	0.53	0.56
9 6 6 2 1 3 1 1 1 1 1 1 1	0	0.1	0.1	1	1	0	0.1	0	0.1	0.21	0.26
2 31 3 4 2 1 3 2 4 2 4 1 2	2	3	2	1	1	0.1	1	0	0	0.00	0.00
3 24 4 2 2 2 3 2 1 4 3 1 2	0	0	0	0	0	1	1	0	0	0.00	0.00
2 30 3 3 3 3 3 3 3 4 3 2 1	0	0	0	0	0	0	0	0.01	0	0.28	0.43
2 30 3 3 3 3 3 3 1 4 3 1 1	1	0	0.01	0	0	0	1	0	0	0.00	0.00
2 31 3 4 2 1 3 3 1 4 2 1 2	2	2	0.1	0.1	0.1	0	0	0.1	3	0.62	0.78
1 39 3 4 2 1 3 2 1 4 3 1 2	0	0.1	2	0	0	2	1	0	0	0.00	0.00
9 8 6 2 3 3 5 1 1 1 1 1 1	1	1	0.1	0	1	0	0	0	0	0.00	0.00
1 34 3 4 2 1 4 2 3 4 3 1 2	0	0	0.1	0	0	0	0	0	0	0.00	0.00
1 39 3 4 2 1 3 2 3 4 3 1 3	0	2	0	0	0	0	0	0	0	0.00	0.00
2 31 3 4 2 1 3 2 1 3 2 1 2	2	1	1	3	2	1	1	3	0	0.67	0.72
2 31 3 4 2 1 3 2 3 3 2 1 2	1	1	1	0	2	0	0	0	0	0.00	0.00
2 30 3 3 3 3 3 2 4 4 2 1 1	0.1	1	0.1	0	0	0	0	0	0	0.00	0.00
2 30 3 3 3 3 3 3 3 2 2 1 1	1	0	0	0	0	0	0	0	0	0.00	0.00
2 31 3 3 2 3 4 3 4 3 3 1 2	0	0	0	0	1	0	0.1	0.1	1	0.42	0.63
4 27 4 2 1 2 3 2 3 4 3 1 2	0	0	0	0	0	0	0	0	0.1	0.30	0.32
3 26 4 2 1 2 3 2 3 6 5 1 2	2	2	1	0.01	0	0	0	0	0	0.00	0.00
3 24 4 3 2 2 3 2 1 4 5 1 2	0.01	1	0.1	0.01	0	0	0	0	0	0.00	0.00
4 27 4 2 1 2 3 2 4 4 4 1 1	1	1	0.1	0	0	0	0	0	0	0.00	0.00
2 23 4 2 2 2 3 2 1 3 3 1 1	0.01	0.1	0.1	0	0	0	0	0	0	0.00	0.00
3 20 5 2 1 2 2 3 4 3 5 1 2	1	2	2	1	1	0	0.1	0.01	0	0.20	0.30
3 20 5 2 1 2 2 3 4 2 3 1 1	0.1	0.1	0	0	1	0.1	0	0.1	0.01	0.19	0.29
3 20 5 2 2 2 2 3 1 1 4 1 2	1	0.1	0	1	2	0	0	0	0	0.00	0.00

Characters (see Table 1)	1 2 3 4 5 6 7 8 9 10 11 12 13
$v_{1,4}$	1 39 3 4 2 1 3 2 1 4 3 1 3
$v_{1,5}$	9 6 6 2 1 3 1 1 1 1 1 1 1
Other relevant information includes:	
Character type (2 qualit., 3 quant.):	2 2 2 2 2 2 2 2 2 3 3 3 3
State ranges (within the data set):	5 4 1 2
Character weight:	1 1 1 1 1 1 1 1 1 1 1 1 1

$$S_{1,5;1,4} = [(8)(1)(0) + (1)(1) + (1)(1 - |4-1|/5) + (1)(1 - |3-1|/4) + (1)(1 - |1-1|/1) + (1)(1- |3-1|/2] / 13 = 0.223.$$

We note that $S_{1,5;1,4} = S_{1,4;1,5}$. Fuzzy adjusted cover-abundances for the community pair 8,9 are presented in Table 2. The matrix of fuzzy adjusted nominal correlations

$$\left(r_{i\alpha\beta} = \frac{Q_{i\alpha\beta}}{\sqrt{Q_{i\alpha\alpha}\,Q_{i\beta\beta}}}\right)$$

is given in Table 3.

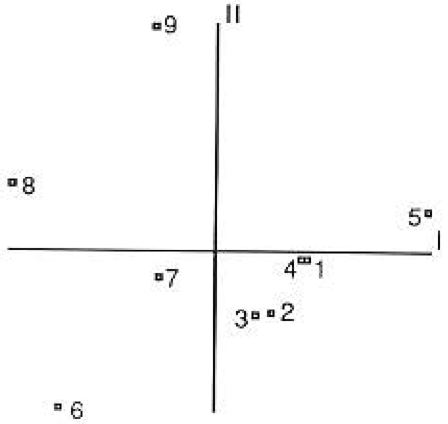

Fig. 1. Scattergram of the first two eigenaxes of the matrix of fuzzy-adjusted nominal correlations (Table 3), hierarchy level 1. The eigenvalues for I and II account respectively for 49.8% and 32.8% of the total. Coherence between vegetation structure so defined and environmental structure (see main text for method) is 0.660 based on the 2 axes, and 0.650 based on all eigenaxes.

Figure 1 displays results emitting from a Q-type eigen-ordination based on nominal correlations[2] (Orlóci and Orlóci 1985) after fuzzy adjustments. Table 4 displays unadjusted nominal correlation coefficients, along with values of the indeterminacy indexes. Figure 2 displays the ordination scattergram based on this set of correlations. Quadrat 9 appears as an extreme outlier in the ordination based on unadjusted nominal correlation coefficients.

Table 3. Nominal correlations after pairwise fuzzy adjustments, hierarchy level 1, based on the Elk Lake data. The method is described in the main text.

Quadrats	1	2	3	4	5	6	7	8	9
1	1	0.998	0.996	0.995	0.995	0.981	0.992	0.981	0.983
2		1	0.999	0.994	0.991	0.986	0.992	0.983	0.980
3			1	0.995	0.992	0.989	0.994	0.985	0.980
4				1	0.997	0.984	0.995	0.979	0.984
5					1	0.966	0.985	0.965	0.980
6						1	0.993	0.986	0.970
7							1	0.990	0.986
8								1	0.989
9									1

Table 4. Unadjusted nominal correlations and respective values of the indeterminacy index, hierarchy level 1, based on the Elk Lake data. The method is described in the main text.

Quadrats	1	2	3	4	5	6	7	8	9
Nominal correlations:									
1	1	0.820	0.644	0.469	0.605	0.136	0.317	0.398	0.365
2		1	0.708	0.347	0.471	0.091	0.246	0.182	0.321
3			1	0.424	0.506	0.410	0.419	0.251	0.024
4				1	0.720	0.529	0.685	0.723	0.045
5					1	0.289	0.434	0.458	0.104
6						1	0.808	0.337	0.020
7							1	0.337	0.031
8								1	0.043
9									1
Indeterminacy index:									
1	0	0.27	0.33	0.53	0.48	0.8	0.73	0.75	0.86
2		0	0.32	0.5	0.38	0.76	0.70	0.77	0.82
3			0	0.58	0.59	0.85	0.65	0.86	0.90
4				0	0.5	0.77	0.57	0.69	0.77
5					0	0.75	0.59	0.6	0.67
6						0	0.55	0.7	0.8
7							0	0.69	0.77
8								0	0.56
9									0

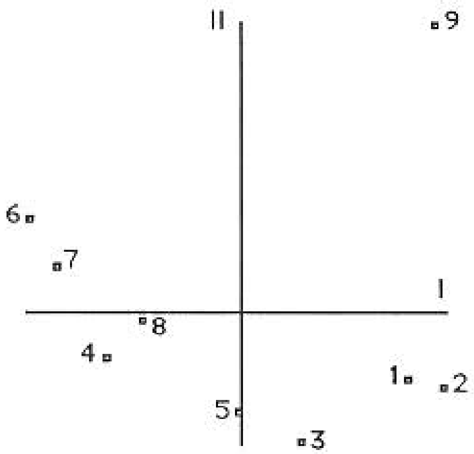

Fig. 2. Scattergram of the first two eigenaxes of the matrix of unadjusted nominal correlations (Table 4), hierarchy level 1. The eigenvalues for I and II account respectively for 34.6% and 22.3% of the total. Coherence between vegetation structure so defined and environmental structure (see main text for method) is 0.125 based on the 2 axes, and 0.304 considering all eigenaxes.

What is the effect of the indeterminacy upon the ecological reliability of the resemblance measure in describing vegetation structure, and how much improvement results from fuzzy set adjustments? The answer can be given in relative terms based on comparing the vegetation structure defined by unadjusted resemblances with the one defined after fuzzy adjustments in terms of coherence with an underlying environmental structure. The method is similar to the one used to evaluate structural stability of mappings in process sampling (Orlóci and Pillar 1989 and Chapter 4). In these terms, a $v \times v$ symmetric matrix **D** of quadrat distances, calculated from nominal correlations using $d_{\alpha\beta} = \sqrt{2(1 - r_{\alpha\beta})}$, or calculated from ordination scores, defines vegetation structure. Another $v \times v$ matrix Δ of quadrat distances based on environmental variables defines environmental structure. The environmental variables considered are elevation, exposure, slope, soil depth, and soil texture. The coherence $\rho(D;\Delta)$ between structures is a product moment correlation involving the $v\ (v - 1)\ /\ 2$ off-diagonal elements in the half distance matrices. This coherence is higher when a fuzzy adjustment is adopted (see Figs. 1 and 2).

Acknowledgements

The authors are recipients of a CNPq (Brazil) grant (V.D.P.P.) and an NSERC (CANADA) grant (L.O.).

Notes

1. For completeness we give

$$S_{ide} = \frac{\sum_{h=1}^{m-i+1} \omega_h t_{ideh} \delta_{ideh}}{\sum_{j=1}^{m-i+1} \delta_{idej}}$$

where ω_h is an arbitrary weight assigned to the hth character in the 0 to 1 interval. t_{ideh} and δ_{ideh} are scores assigned according to the type of the hth character as follows:

a) If character h is dichotomous (the states are presence or absence; +,-):

v_{idh}	+	+	−	−
v_{ieh}	+	−	+	−
t_{ideh}	1	0	0	0
δ_{ideh}	1	1	1	0

where v_{idh} and v_{ieh} are members of vectors v_{id} and v_{ie}, that is, the states of the defining character h.

b) If character h is qualitative, $t_{ideh} = 1$ if v_{idh} and v_{ieh} agree, and $t_{ideh} = 0$ if v_{idh} and v_{ieh} disagree. In either case $\delta_{ideh} = 1$.

c) If character h is quantitative

$$t_{ideh} = 1 - \frac{|v_{idh} - v_{ieh}|}{(\max v_{ih} - \min v_{ih})}.$$

The extreme values for character h may be defined *a priori* or as the ones realized within the sample. In all cases $\delta_{ideh} = 1$.

2. A matrix of distances **D**, is derived from the r values based on $d_{\alpha\beta} = \sqrt{2(1 - r_{\alpha\beta})}$ which in turn is subjected to D-type eigenanalysis (Wildi and Orlóci 1990).

References

Barkman, J.J. 1988. New systems of plant growth forms and phenological plant types. *In:* M.J.A. Werger, P.J.M. van der Aart, H.J. During and J.T.A. Verhoeven (eds.) *Plant Form and Vegetation Structure; Adaptation, Plasticity and Relation to Herbivory.* SPB Academic Publishing, The Hague.

Dale, M. 1988. Some fuzzy approaches to phytosociology: ideals and instances. *Folia Geobotanica et Phytotaxonomica* 23: 239–274.

Dansereau, P. 1957. *Biogeography; An Ecological Perspective.* Ronald Press, New York.

Feoli, E. and V. Zuccarello. 1986. Ordination based on classification: yet another solution? *Abstracta Botanica* 10: 203–219.

Feoli, E. and V. Zuccarello. 1988. Syntaxonomy: a source of useful fuzzy sets for environmental analysis? *Coenoses* 3: 141–147.

Gower, J.C. 1971. A general coefficient of similarity and some of its properties. *Biometrics* 27: 857–871.

Marsili-Libelli, S. 1989. Fuzzy clustering of ecological data. *Coenoses* 4: 95–106.

Mueller-Dombois, D. and H. Ellenberg. 1974. *Aims and Methods of Vegetation Ecology*. John Wiley & Sons, New York.

Orlóci, L. 1988. Detecting vegetation patterns. *ISI Atlas of Science* 1: 173–177.

Orlóci, L. 1991 On character-based plant community analysis: choice, arrangement, comparison. (see Chapter 8).

Orlóci, L., E. Feoli, D. Lausi and P. Nimis. 1986. Estimation of character structure convergence (divergence) in plant communities: a nested hierarchical model. *Coenoses* 1: 11–20.

Orlóci, L. and M. Orlóci. 1985. Comparison of communities without the use of species: model and example. *Annali di Botanica* 43: 275–285.

Orlóci, L. and V. De Patta Pillar. 1989. On sample size optimality in ecosystem survey. *Biometrie-Praximetrie* 29: 173–184.

Orlóci, L. and S.L. Stofella. 1986. The taxon-free approach to the study of plant communities. *Annals of Arid Zone* 25: 111–131.

Roberts, D.W. 1986. Ordination on the basis of fuzzy set theory. *Vegetatio* 66: 123–131.

Wildi, O. and L. Orlóci. 1990. *Numerical Exploration of Community Patterns*. SPB Academic Publishing, The Hague.

Shreve, F. 1942. The desert vegetation of North America. *Botanical Review* 8: 195–246.